ANOVA for functional time series data: when there is dependence between groups

Siegfried Hörmann
Institute of Statistics
Graz University of Technology

Functional time series (FTS) have become a relatively popular area of research during the past years. Examples of FTS's include annual temperature or smoothed precipitation curves, daily pollution level curves, various daily curves derived from high frequency asset price data, daily bond yield curves, daily vehicle traffic curves and many others.

In this talk we discuss a very central topic in time series analysis, which is finding a potential periodic pattern in our data. In fact, tests for periodicity go back to the very origins of the field. The importance of such tests has manifold reasons. One of them is that most inferential procedures require that the series be stationary, but classical stationarity tests (as e.g. KPSS procedures) have little power against a periodic component in the mean. Not surprisingly, this topic is equally relevant for functional time series data.

Obviously, a period of length d defines a factor with d levels. So this naturally leads to an analysis of variance (ANOVA) problem. The main emphasis of this talk lies then on the development of a *fully functional ANOVA test for stationary data*. If the functional time series (Y_t) satisfies a certain weak-dependence condition, then, using a frequency domain approach, we obtain the asymptotic null-distribution (for the constant mean hypothesis) of the functional ANOVA statistic. The limiting distribution has an interesting form and can be written as a sum of independent hypoexponential variables whose parameters are eigenvalues of the spectral density operator of (Y_t). To the best of our knowledge, there exists no comparable asymptotic result in FDA literature.

A common approach to inference for functional data is to project observations onto a low dimensional basis system and then to apply a suitable multivariate procedure to the vector of projections. This approach will also be explained and discussed.

The talk is based on joint work with Piotr Kokoszka (Colorado State University) and Gilles Nisol (ULB).