Widely Linear Data Estimation for Unique Word OFDM

Mario Huemer, *Senior Member, IEEE*, Alexander Onic, *Student Member, IEEE*,
Christian Hofbauer, *Student Member, IEEE*, Stefan Trampitsch
Johannes Kepler University Linz, Institute of Signal Processing
Altenbergerstr. 69, 4040 Linz, Austria {mario.huemer, alexander.onic, christian.hofbauer}@jku.at

***Abstract*—Unique word - orthogonal frequency division multiplexing (UW-OFDM) is known to feature an excellent bit error ratio performance when compared to conventional OFDM using cyclic prefixes (CP). In recent papers linear and non-linear UW-OFDM receivers have been investigated and compared for proper data vectors consisting of QPSK and 16-QAM symbols. In this work we derive the widely linear minimum mean square error (WLMMSE) estimator for the application to UW-OFDM in combination with real and therefore improper data vectors. For the investigated modulation schemes BPSK and M-ASK our simulation results indicate a significant performance gain of the widely linear estimator over strictly linear methods.**

***Index Terms*—Cyclic Prefix (CP), Unique Word (UW), UW-OFDM, non-systematically encoded UW-OFDM, LMMSE, WLMMSE.**

I. Introduction

In [1]-[3] we introduced a block-based OFDM-like signaling scheme, where the guard intervals are filled with a deterministic sequence - the unique word (UW). For the generation of the unique words we introduced so-called redundant subcarriers, see also [4]-[7]. In [8] the concept has been generalized by allowing the redundancy to be spread over all subcarriers. The resulting non-systematically encoded UW-OFDM systems clearly outperform the originally introduced systematically encoded approaches. At the price of higher complexity the performance of the linear data estimators, studied e.g. in [3], [8], can be further improved by non-linear estimation approaches like sphere decoding, cf. [9]. Besides the higher complexity most of the known non-linear estimation schemes have the drawback, that soft information to be used for channel decoding is very difficult to compute. Usually only approximations can reasonably be derived, cf. [10]. However, when real and therefore improper data vectors consisting of BPSK or M-ASK symbols are used, the linear minimum mean square error (LMMSE) estimator can also be outperformed by the widely linear MMSE (WLMMSE) data estimator. Like the strictly linear counterparts, WLMMSE estimation has the nice feature that a closed form solution of the error covariance matrix exists, such that soft information for channel decoding can easily be obtained.

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The paper is organized as follows: In Section II we introduce the basics of UW-OFDM, and we discuss the systematic as well as the non-systematic symbol generation approaches. The system model and linear data estimators are described in Section III. In Section IV the WLMMSE estimator for real data vectors is introduced, and finally simulation results and performance discussions are given in Section V.

II. Unique Word OFDM Basics

Let $\mathbf{x}_u \in \mathbb{C}^{N_u \times 1}$ be a predefined sequence which we call unique word. This unique word shall form the tail of each time domain OFDM symbol vector. Hence, an OFDM time domain symbol vector consists of two parts and is of the form $\mathbf{x}' = \begin{bmatrix}\mathbf{x}_d^T & \mathbf{x}_u^T\end{bmatrix}^T \in \mathbb{C}^{N\times 1}$. In the concept suggested in [1], [2] we generate an UW-OFDM symbol $\mathbf{x} = \begin{bmatrix}\mathbf{x}_d^T & \mathbf{0}^T\end{bmatrix}^T$ with a zero UW in a first step, and determine the final transmit symbol $\mathbf{x}' = \mathbf{x} + \begin{bmatrix}\mathbf{0}^T & \mathbf{x}_u^T\end{bmatrix}^T$ by adding the desired UW in time domain in a second step. As in conventional OFDM, the QAM data symbols (denoted by the vector $\mathbf{d} \in \mathbb{C}^{N_d\times 1}$) and the zero subcarriers (usually at the band edges and at DC) are specified in frequency domain as part of the vector $\tilde{\mathbf{x}}$, but here in addition the zero-word is specified in time domain as part of the vector $\mathbf{x} = \mathbf{F}_N^{-1}\tilde{\mathbf{x}}$. Here, $\mathbf{F}_N$ denotes the length-N-DFT matrix with elements $[\mathbf{F}_N]_{k,l} = \mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}kl}$ for $k,l = 0,1,...,N-1$. The system of equations $\mathbf{x} = \mathbf{F}_N^{-1}\tilde{\mathbf{x}}$ with the introduced features can be fulfilled by introducing redundancy in the frequency domain. We do that by defining codewords

$$\mathbf{c} = \mathbf{G}\mathbf{d} \tag{1}$$

with the help of appropriate complex valued generator matrices $\mathbf{G} \in \mathbb{C}^{(N_d+N_r)\times N_d}$, where $N_r = N_u$. The frequency domain symbol $\tilde{\mathbf{x}}$ is finally built by inserting the zero-subcarriers which can be modeled by $\tilde{\mathbf{x}} = \mathbf{B}\mathbf{c}$, where $\mathbf{B} \in \{0,1\}^{N\times(N_d+N_r)}$ consists of zero-rows at the positions of the zero subcarriers, and of appropriate unit row vectors at all other positions. To produce the zero-UW in time domain a valid code generator matrix has to fulfill

$$\mathbf{F}_N^{-1}\mathbf{B}\mathbf{G} = \begin{bmatrix}* \\ \mathbf{0}\end{bmatrix}. \tag{2}$$

Note that $\mathbf{F}_N^{-1}\mathbf{B}$ is composed of those columns of $\mathbf{F}_N^{-1}$ that correspond to the non-zero entries of the OFDM frequency

domain symbol $\tilde{\mathbf{x}}$. Let $\mathbf{W} \in \mathbb{C}^{N_r \times (N_d+N_r)}$ be the matrix built by the N_r lowermost rows of $\mathbf{F}_N^{-1}\mathbf{B}$. Then the constraint (2) can also be formulated as

$$\mathbf{WG} = \mathbf{0}, \tag{3}$$

which says that the columns of a valid $\mathbf{G}$ have to lie in the null space of $\mathbf{W}$.

Since we focus on data estimation and not on system parameter estimation or synchronization tasks in this paper the particular shape of the UW is irrelevant for the investigations below. We therefore assume $\mathbf{x}_u = \mathbf{0}$ from here on.

A. Systematically Encoded UW-OFDM

In our original UW-OFDM concept presented in [1]-[3] we chose

$$\mathbf{G} = \mathbf{P}\begin{bmatrix}\mathbf{I}\\ \mathbf{T}\end{bmatrix}, \tag{4}$$

where $\mathbf{P} \in \{0,1\}^{(N_d+N_r)\times(N_d+N_r)}$ is a (carefully selected) permutation matrix, cf. [4]-[7], [9]. Let $\mathbf{M} = \mathbf{WP} = \begin{bmatrix}\mathbf{M}_1 & \mathbf{M}_2\end{bmatrix}$ with $\mathbf{M}_1 \in \mathbb{C}^{N_r \times N_d}$ and $\mathbf{M}_2 \in \mathbb{C}^{N_r \times N_r}$, then the constraint (3) is fulfilled by choosing $\mathbf{T} = -\mathbf{M}_2^{-1}\mathbf{M}_1 \in \mathbb{C}^{N_r \times N_d}$. We call $\mathbf{r} = \mathbf{Td} \in \mathbb{C}^{N_r \times 1}$ the vector of redundant symbols. Hence, a codeword can be written as

$$\mathbf{c} = \mathbf{P}\begin{bmatrix}\mathbf{d}\\ \mathbf{r}\end{bmatrix}. \tag{5}$$

Consequently, this approach leads to codewords $\mathbf{c}$ with dedicated data and dedicated redundant elements. We therefore refer to this approach as systematically encoded UW-OFDM.

B. Non-Systematically Encoded UW-OFDM

In [8] we introduced the concept of non-systematically encoded UW-OFDM, where we propose code generator matrices $\mathbf{G}$ that distribute the redundancy over all subcarriers. We suggested to derive optimum code generator matrices by minimizing the trace of the error covariance matrices of the best linear unbiased estimator (BLUE) and the LMMSE estimator, respectively, for the case $\tilde{\mathbf{H}} = \mathbf{I}$ (that is the AWGN channel case) and for a fixed signal-to-noise ratio. In addition the constraint (3) has to be fulfilled. It has been shown, that both constrained optimization problems (for the BLUE and for the LMMSE estimator) are globally solved by a matrix $\mathbf{G}$ if and only if $\mathbf{G}$ fulfills (3) and

$$\mathbf{G}^H\mathbf{G} = s^2\mathbf{I}, \tag{6}$$

where s are the all identical singular values of $\mathbf{G}$. It is equivalent to say that a matrix is an optimum code generator matrix if and only if the columns of $\mathbf{G}$ build an orthogonal basis of the nullspace of $\mathbf{W}$. In the following, we only consider normalized optimum code generator matrices such that $s^2 = 1$ or $\mathbf{G}^H\mathbf{G} = \mathbf{I}$. This normalization implies that the operation $\mathbf{c} = \mathbf{Gd}$ becomes energy-invariant. In [8] we found optimum code generator matrices by choosing

$$\mathbf{G} = \mathbf{A}\begin{bmatrix}\mathbf{I}\\ \mathbf{T}\end{bmatrix}, \tag{7}$$

with a non-singular $\mathbf{A} \in \mathbb{R}^{(N_d+N_r)\times(N_d+N_r)}$. Let $\mathbf{M}$ now be $\mathbf{M} = \mathbf{WA} = \begin{bmatrix}\mathbf{M}_1 & \mathbf{M}_2\end{bmatrix}$, then the constraint (3) is again fulfilled by choosing

$$\mathbf{T} = -\mathbf{M}_2^{-1}\mathbf{M}_1. \tag{8}$$

The optimization problems can therefore be treated as unconstrained problems in $\mathbf{A}$, which can numerically be solved e.g. with the steepest descent method. Different generator matrices can be found by using different initializations for $\mathbf{A}$. In the simulations of this work we use a generator matrix which has been derived by a random initialization of $\mathbf{A}$. This particular generator matrix, which has been discussed in detail in [8], is denoted as $\mathbf{G}''$.

III. System Model and Linear Data Estimators

As derived in detail in [1]-[3], after subtraction of a UW depending portion (which becomes $\mathbf{0}$ for $\mathbf{x}_u = \mathbf{0}$), the receive symbol $\tilde{\mathbf{y}} \in \mathbb{C}^{(N_d+N_r)\times 1}$ (already excluding the zero subcarrier symbols) can be modeled as

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\mathbf{Gd} + \tilde{\mathbf{v}}, \tag{9}$$

where the diagonal matrix $\tilde{\mathbf{H}} \in \mathbb{C}^{(N_d+N_r)\times(N_d+N_r)}$ contains the corresponding channel frequency response coefficients on its main diagonal, and $\tilde{\mathbf{v}}$ represents a zero-mean Gaussian (frequency domain) noise vector with the covariance matrix $\mathbf{C}_{vv} = \sigma_v^2\mathbf{I}$. Furthermore, we assume the data vector to have zero mean and covariance matrix $\mathbf{C}_{dd} = \sigma_d^2\mathbf{I}$.

In [3], [8] linear data estimators of the form $\hat{\mathbf{d}} = \mathbf{E}\tilde{\mathbf{y}}$ have been derived and studied. A simple zero forcing (ZF) solution for systematically encoded UW-OFDM is the channel inversion estimator given by

$$\mathbf{E}_{\text{CI}} = \begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{P}^T\tilde{\mathbf{H}}^{-1}. \tag{10}$$

For both systematically and non-systematically encoded UW-OFDM the optimum ZF data estimator corresponds to the BLUE as a representative of classical estimators given by

$$\mathbf{E}_{\text{BLUE}} = (\mathbf{G}^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{G})^{-1}\mathbf{G}^H\tilde{\mathbf{H}}^H. \tag{11}$$

Its covariance matrix $\mathbf{C}_{\hat{d}\hat{d}}$ coincides with the covariance matrix $\mathbf{C}_{ee}$ of the error vector $\mathbf{e} = \hat{\mathbf{d}}_{\text{BLUE}} - \mathbf{d}$ and is given by

$$\mathbf{C}_{ee} = \sigma_v^2(\mathbf{G}^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{G})^{-1}. \tag{12}$$

The Bayesian LMMSE estimator and the covariance matrix of its error vector $\mathbf{e} = \hat{\mathbf{d}}_{\text{LMMSE}} - \mathbf{d}$ are given by $\mathbf{E}_{\text{LMMSE}} = \mathbf{C}_{dy}\mathbf{C}_{yy}^{-1}$ and $\mathbf{C}_{ee} = \mathbf{C}_{dd} - \mathbf{C}_{dy}\mathbf{C}_{yy}^{-1}\mathbf{C}_{dy}^H$, respectively. For uncorrelated data with the covariance matrix $\mathbf{C}_{dd} = \sigma_d^2\mathbf{I}$ the estimator and its error covariance matrix take on the form

$$\mathbf{E}_{\text{LMMSE}} = (\mathbf{G}^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{G} + \tfrac{\sigma_v^2}{\sigma_d^2}\mathbf{I})^{-1}\mathbf{G}^H\tilde{\mathbf{H}}^H, \tag{13}$$

and

$$\mathbf{C}_{ee} = \sigma_v^2(\mathbf{G}^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{G} + \tfrac{\sigma_v^2}{\sigma_d^2}\mathbf{I})^{-1}, \tag{14}$$

respectively. We note that the expectation operation for deriving the error covariance matrix in (14) is with respect to the joint probability density function (PDF) of $\tilde{\mathbf{y}}$ and $\mathbf{d}$ (Bayesian

approach), while for deriving the error covariance matrix in (12) the expectation operation is with respect to the PDF of $\tilde{\mathbf{y}}$ only, since $\mathbf{d}$ is assumed to be deterministic (classical approach).

IV. Widely Linear Data Estimation

A data vector fulfills the properness condition, cf. [11], if $\tilde{\mathbf{C}}_{dd} = E[\mathbf{d}\mathbf{d}^T] = \mathbf{0}$. $\tilde{\mathbf{C}}_{dd}$ is called complementary or pseudo data covariance matrix. The properness condition is fulfilled by data vectors consisting of e.g. QPSK, 16-QAM or 64-QAM symbols. For improper data vectors, i.e. $\tilde{\mathbf{C}}_{dd} \neq \mathbf{0}$, the *strictly* linear MMSE estimator can be outperformed by the *widely* linear MMSE estimator which is given by

$$\hat{\mathbf{d}}_{\text{WLMMSE}} = \mathbf{E}_1\tilde{\mathbf{y}} + \mathbf{E}_2\tilde{\mathbf{y}}^* \tag{15}$$

for zero mean data vectors. For the determination of the WLMMSE estimator we not only require the covariance matrix $\mathbf{C}_{yy} = E[\tilde{\mathbf{y}}\tilde{\mathbf{y}}^H]$ and the cross covariance matrix $\mathbf{C}_{dy} = E[\mathbf{d}\tilde{\mathbf{y}}^H]$, but in addition the complementary covariance matrix $\tilde{\mathbf{C}}_{yy} = E[\tilde{\mathbf{y}}\tilde{\mathbf{y}}^T]$ and the complementary cross covariance matrix $\tilde{\mathbf{C}}_{dy} = E[\mathbf{d}\tilde{\mathbf{y}}^T]$. The estimator matrices in (15) are given by

$$\mathbf{E}_1 = (\mathbf{C}_{dy} - \tilde{\mathbf{C}}_{dy}\mathbf{C}_{yy}^{-*}\tilde{\mathbf{C}}_{yy}^*)\mathbf{P}_{yy}^{-1}, \tag{16}$$

$$\mathbf{E}_2 = (\tilde{\mathbf{C}}_{dy} - \mathbf{C}_{dy}\mathbf{C}_{yy}^{-1}\tilde{\mathbf{C}}_{yy})\mathbf{P}_{yy}^{-*}, \tag{17}$$

where $\mathbf{P}_{yy} = \mathbf{C}_{yy} - \tilde{\mathbf{C}}_{yy}\mathbf{C}_{yy}^{-*}\tilde{\mathbf{C}}_{yy}^*$, cf. [11]. The covariance matrix $\mathbf{C}_{ee}$ of the error vector $\mathbf{e} = \hat{\mathbf{d}}_{\text{WLMMSE}} - \mathbf{d}$ reads as

$$\begin{aligned}\mathbf{C}_{ee} &= \mathbf{C}_{dd} - (\mathbf{C}_{dy} - \tilde{\mathbf{C}}_{dy}\mathbf{C}_{yy}^{-*}\tilde{\mathbf{C}}_{yy}^*)\mathbf{P}_{yy}^{-1}\mathbf{C}_{dy}^H \\ &\quad -(\tilde{\mathbf{C}}_{dy} - \mathbf{C}_{dy}\mathbf{C}_{yy}^{-1}\tilde{\mathbf{C}}_{yy})\mathbf{P}_{yy}^{-*}\tilde{\mathbf{C}}_{dy}^H.\end{aligned} \tag{18}$$

For proper data vectors the complementary covariance matrices $\tilde{\mathbf{C}}_{dd}, \tilde{\mathbf{C}}_{yy}$ and the complementary cross-covariance matrix $\tilde{\mathbf{C}}_{dy}$ become zero matrices, and the WLMMSE estimator degenerates to the LMMSE estimator, i.e. $\mathbf{E}_1 = \mathbf{E}, \mathbf{E}_2 = \mathbf{0}$.

A. Widely Linear Data Estimator for Real Data Vectors

In this paper we focus on data vectors consisting of elements out of real alphabets like e.g. BPSK and 8-ASK, which are improper by nature. For such data vectors and under the assumption of uncorrelated data symbols the data covariance matrix and the complementary data covariance matrix become $\mathbf{C}_{dd} = \tilde{\mathbf{C}}_{dd} = \sigma_d^2\mathbf{I}$. For the complementary cross-covariance matrix we have $\tilde{\mathbf{C}}_{dy} = \mathbf{C}_{dy}^*$, and the WLMMSE estimator (15)-(17) simplifies to $\mathbf{E}_2 = \mathbf{E}_1^*$ or

$$\hat{\mathbf{d}}_{\text{WLMMSE}} = 2\text{Re}\{\mathbf{E}_1\tilde{\mathbf{y}}\}. \tag{19}$$

While the LMMSE estimate of a real data vector from a complex receive vector is generally complex, the WLMMSE estimate is always real. Also the error covariance matrix (18) simplifies and can be written as

$$\mathbf{C}_{ee} = \mathbf{C}_{dd} - 2\text{Re}\{(\mathbf{C}_{dy} - \tilde{\mathbf{C}}_{dy}\mathbf{C}_{yy}^{-*}\tilde{\mathbf{C}}_{yy}^*)\mathbf{P}_{yy}^{-1}\mathbf{C}_{dy}^H\}. \tag{20}$$

By inserting the linear model equation (9) the required covariance and complementary covariance matrices become

$$\mathbf{C}_{yy} = \sigma_d^2\tilde{\mathbf{H}}\mathbf{G}\mathbf{G}^H\tilde{\mathbf{H}}^H + \sigma_v^2\mathbf{I}, \tag{21}$$

$$\tilde{\mathbf{C}}_{yy} = \sigma_d^2\tilde{\mathbf{H}}\mathbf{G}\mathbf{G}^T\tilde{\mathbf{H}}^T, \tag{22}$$

$$\mathbf{C}_{dy} = \sigma_d^2\mathbf{G}^H\tilde{\mathbf{H}}^H, \tag{23}$$

$$\tilde{\mathbf{C}}_{dy} = \sigma_d^2\mathbf{G}^T\tilde{\mathbf{H}}^T. \tag{24}$$

B. Optimum Code Generator Matrices for Real Data Vectors

In [14] we show that fulfilling (3) together with (6) is again sufficient for a code generator matrix to perform optimum for real data vectors in combination with WLMMSE estimation under AWGN conditions. However, the condition is not necessary. Let $\mathbf{G}_0 = \begin{bmatrix}\mathbf{g}_0 & \mathbf{g}_1 & \mathbf{g}_2 & \cdots & \mathbf{g}_{N_d-1}\end{bmatrix}$ be a code generator matrix fulfilling $\mathbf{G}_0^H\mathbf{G}_0 = \mathbf{I}$ and $\mathbf{W}\mathbf{G}_0 = \mathbf{0}$. Then it can be shown that any matrix built by an arbitrary subset of N_d vectors out of the $2N_d$ vectors $\{\mathbf{g}_0, j\mathbf{g}_0, \mathbf{g}_1, j\mathbf{g}_1, \cdots, \mathbf{g}_{N_d-1}, j\mathbf{g}_{N_d-1}\}$ forms a new code generator matrix featuring the same MSE performance in the AWGN channel as the original matrix $\mathbf{G}_0$. The new generator matrix does not necessarily have to fulfill (6).

V. Simulation Results

In the following we show simulation results for systematically as well for non-systematically encoded UW-OFDM. The most important parameters of our simulated system approaches are specified in Table I.

TABLE I
Main PHY parameters of the investigated UW-OFDM systems.

Modulation schemes	BPSK, 8-ASK, QPSK
Coding rates (outer code)	1/2
FFT length N	64
Occupied subcarriers	52
Number of zero-subcarriers	12
Length of data vector N_d	36
Length of guard interval N_u	16
Sampling frequency	20 MHz

As in [12] the indices of the zero subcarriers within an OFDM symbol $\tilde{\mathbf{x}}$ are set to {0, 27, 28,...,37}. For the systematically encoded UW-OFDM system the indexes of the redundant subcarriers are chosen to be {2, 6, 10, 14, 17, 21, 24, 26, 38, 40, 43, 47, 50, 54, 58, 62}, cf. [1]. Since we focus on data estimation procedures in this work rather than on synchronization or channel estimation approaches we chose the zero UW for the bit error ratio (BER) simulations below. For outer channel coding we used the same convolutional encoder with the industry standard rate 1/2, constraint length 7 code with generator polynomials (133, 171) as defined in [12]. A soft decision Viterbi algorithm is applied for decoding. The main diagonal of the appropriate error covariance matrix $\mathbf{C}_{ee}$ is used to specify the varying noise variances along the data symbols after data estimation.

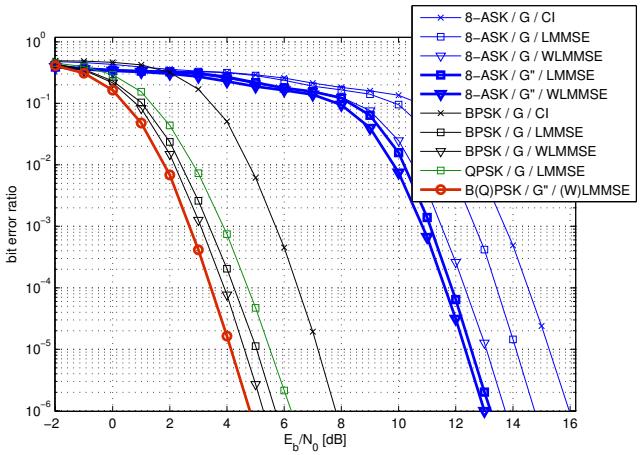

Fig. 1. BER simulation results in the AWGN channel.

A. Simulation Results in the AWGN Channel

Clearly, OFDM systems are designed to overcome the problems of frequency selective channels. However, we start with simulations in the AWGN channel, since these results provide first interesting insights. In the following the particular code generator matrices for the systematically encoded and the non-systematically encoded UW-OFDM system are denoted by $\mathbf{G}$ and $\mathbf{G}''$, respectively.

Fig. 1 shows BER over E_b/N_0 simulation results for BPSK and 8-ASK as well as for QPSK as a reference. BPSK is used as a fall back solution in many standards and constitutes a slow but reliable operation mode. 8-ASK is rarely used in practice, however we include the results for demonstration purposes. In the following all performance comparisons are measured at a BER of 10^{-6}.

For 8-ASK in combination with $\mathbf{G}$ the WLMMSE receiver outperforms the LMMSE and the CI receiver by 1dB and 2.2 dB, respectively. Another 0.5dB and 0.75dB are gained by $\mathbf{G}''$/LMMSE and $\mathbf{G}''$/WLMMSE, respectively.

The situation is similar for systematically encoded UW-OFDM ($\mathbf{G}$) in combination with BPSK, here the gains of $\mathbf{G}$/WLMMSE over $\mathbf{G}$/LMMSE and $\mathbf{G}$/CI are 0.4dB and 2.5dB, respectively. As a quite interesting result we note that BPSK/$\mathbf{G}$/LMMSE clearly outperforms QPSK/$\mathbf{G}$/LMMSE. This differs to the usual behavior of CP-OFDM or single carrier transmission schemes, where BPSK and QPSK perform equivalently in the AWGN channel, when plotted over E_b/N_0. Eventually, BPSK/$\mathbf{G}$/WLMMSE outperforms the QPSK reference system by around 1dB which is quite remarkable.

The situation is different for non-systematically encoded UW-OFDM ($\mathbf{G}''$). Here, the schemes BPSK/LMMSE, BPSK/WLMMSE and QPSK/LMMSE perform equivalently, outperforming the best performing systematically encoded system (BPSK/WLMMSE) by 0.5 dB. The equivalent performance of the LMMSE and the WLMMSE receiver for BPSK transmission can be explained by inserting $\mathbf{G}^H\mathbf{G} = \mathbf{I}$ and

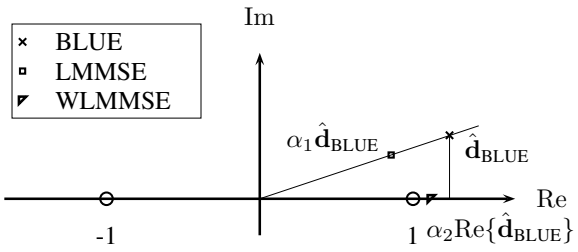

Fig. 2. Relationships between the BLUE, LMMSE, and WLMMSE estimates in the AWGN channel.

$\tilde{\mathbf{H}} = \mathbf{I}$ into (11), (13) and (19) which gives

$$\hat{\mathbf{d}}_{\text{BLUE}} = \mathbf{G}^H\tilde{\mathbf{y}}, \tag{25}$$

$$\hat{\mathbf{d}}_{\text{LMMSE}} = \frac{\sigma_d^2}{\sigma_d^2+\sigma_v^2}\mathbf{G}^H\tilde{\mathbf{y}} = \alpha_1\hat{\mathbf{d}}_{\text{BLUE}}, \tag{26}$$

$$\hat{\mathbf{d}}_{\text{WLMMSE}} = \frac{2\sigma_d^2}{2\sigma_d^2+\sigma_v^2}\text{Re}\{\mathbf{G}^H\tilde{\mathbf{y}}\} = \alpha_2\text{Re}\{\hat{\mathbf{d}}_{\text{BLUE}}\}, \tag{27}$$

with $0 < \alpha_1 \leq 1$, $0 < \alpha_2 \leq 1$, and $\alpha_2 \geq \alpha_1$. Fig. 2 illustrates these relationships for a BPSK constellation: The LMMSE estimate lies on the straight connection of the origin and the BLUE estimate, and the WLMMSE estimator first projects the BLUE estimate onto the real axis and then scales the projected value with α_2. These considerations explain that once the BLUE estimate in BPSK transmission lies in the right (wrong) decision half-plane, then the LMMSE and the WLMMSE estimate will also lie in the right (wrong) decision region. In terms of the Bayesian MSE the WLMMSE outperforms the LMMSE estimator, and the LMMSE estimator performs better than the BLUE, however these gains are not translated into BER gains. The same conclusion is not true for higher order ASK constellations, since the estimates of the BLUE, the LMMSE and the WLMMSE receiver might fall in different decision regions for such schemes, see also Fig. 1.

B. Performance Results in Frequency Selective Indoor Environments

For the generation of indoor multipath channels we applied the model described in [13], which has also been used during the IEEE 802.11a standardization process. The channel impulse responses are modeled as tapped delay lines, each tap with uniformly distributed phase and Rayleigh distributed magnitude, and with power decaying exponentially. The model allows the choice of the channel delay spread. For the following simulations we generated and stored 10000 realizations of channel impulse responses, featuring (on average) a delay spread of 100ns and a total length not exceeding the guard interval duration. The impulse responses have been normalized such that the receive power is independent of the actual channel. The subsequent figures represent BER results averaged over that 10000 channel realizations. We assumed perfect channel knowledge at the receiver in the simulations to be presented below. In our discussion we concentrate on non-systematically encoded

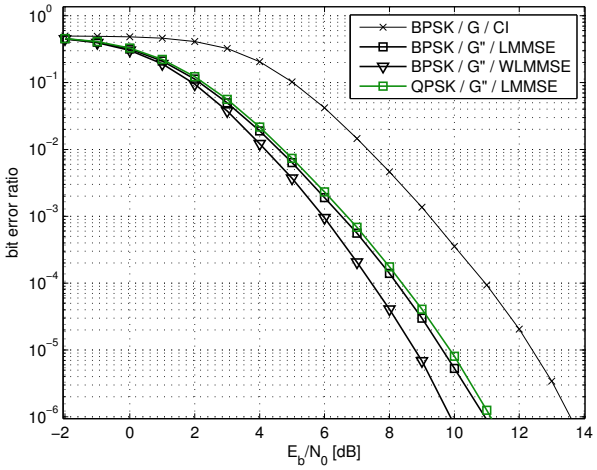

Fig. 3. BER simulation results in frequency selective indoor environments.

UW-OFDM ($\mathbf{G}''$), and we only show one systematically encoded case ($\mathbf{G}$/CI) for comparison reasons. Different to the AWGN results BPSK/$\mathbf{G}''$/WLMMSE, BPSK/$\mathbf{G}''$/LMMSE and QPSK/$\mathbf{G}''$/LMMSE perform quite different in frequency selective scenarios. BPSK/$\mathbf{G}''$/WLMMSE outperforms BPSK/$\mathbf{G}''$/LMMSE by 1dB, and the QPSK mode is outperformed by 1.2dB, such that the combination BPSK/$\mathbf{G}''$/WLMMSE can be considered as an exceptionally well performing fallback mode for the regarded UW-OFDM transmission.

VI. Conclusion

In this work we investigated widely linear data estimation in unique word OFDM for data vectors consisting of elements out of a real symbol alphabet. Especially interesting is BPSK, as this modulation scheme constitutes a low rate fallback solution in many standards. We discussed some interesting differences in estimation performance between systematically and non-systematically encoded UW-OFDM in the AWGN channel. Finally, we showed that for the investigated non-systematic code generator matrix the gain of WLMMSE over LMMSE estimation in frequency selective scenarios is remarkable, and the combination BPSK/WLMMSE can be considered as a highly reliable low rate transmission scheme.

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