Battery state estimation using mixed Kalman/H_∞, adaptive Luenberger and sliding mode observer

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***Abstract*—For electric vehicles, the improvement of the range of miles and with it the utilization of the available cell/battery capacity has become an important research focus in the community. For optimization of the same, an accurate knowledge of internal cell parameters like the state-of-charge (SoC) or the impedance is indispensable. Compared to the state-of-the-art, in this paper discrete-time Kalman and H_∞ filtering based SoC estimation schemes - up to now applied to linear battery models - are applied to the nonlinear model of a Li-Ion battery. For that, a linearization method is proposed, which utilizes a prior knowledge about the predominant nonlinearities in the model together with a coarse SOC estimate to obtain a linear state estimation problem. Based on that, a mixed Kalman/H_∞ filter-, a discrete-time sliding mode observer-, and an adaptive Luenberger based estimation scheme is furthermore investigated for the nonlinear battery model under test. The above-mentioned methods are compared to the state-of-the-art reduced order SoC observer and the Coulomb counting method. In order to compare the performance, an appropriate battery simulation framework is used, which includes measurement and modeling uncertainties. The evaluation is done with respect to the ability to reduce the impact of error sources present in realistic scenarios. For the simulated load current pattern, best results are achieved by the mixed Kalman/H_∞ filtering approach, which achieves an average SoC estimation error of less than 1%.**

I. Introduction

The performance of electrical vehicles (EVs) and hybrid electrical vehicles (HEVs) is strongly influenced by the used battery pack. For optimization with respect to energy utilization, lifetime extension or damage prevention, an accurate and reliable knowledge of internal cell parameters is needed. Especially parameters like the state-of-charge (SoC) or the state-of-health (SoH) are of particular interest. In practice, the SoC is often determined based on the Coulomb counting (CC) method. By simply using the CC method, a highly accurate determination of the SoC cannot necessarily be achieved. Inaccuracies are mainly caused by the accumulation of errors inherently included in the current/charge integration stage. Errors are introduced by the time-variant offset error which is present in a realistic current measurement apparatus. Other error sources are the current sampling process, combined with the uncertainty in the initial SoC. This may significantly degrade the accuracy of the CC method over time [1, 2].

During periods of low loads, typically the CC method is re-initialized by the use of predefined SoC-open-circuit voltage (OCV) tables. In general, these SoC-OCV tables are obtained by characterizing the battery - such that the relation between SoC and OCV is known in advance. To rely on these tables, resting times up to 10 h could be required - mainly depending on factors like the SoC or the ambient temperature. Unfortunately, the availability of such long resting (relaxation) periods is often not given for practical applications. In [3, 4], we proposed an OCV extrapolation method, suitable for predicting the SoC after a short relaxation time. This can be used as a workaround to cope with initial SoC uncertainities.

In literature, many different approaches for the estimation of the internal battery states have been proposed. Examples are the Kalman/H_∞ filter [2, 5], the extended Kalman filter (EKF) [6], the unscented Kalman filter (UKF) [7], the Luenberger- [8] and the sliding mode (SM)- [9, 10] observer. Thereby, the battery model is described as a linear or a nonlinear system.

In this paper, the focus is on the application of selected state estimation schemes for a nonlinear battery system description. In the past, both Kalman and H_∞ filtering have been applied to linear battery models. In Section III, it is shown that Kalman/H_∞ filtering can also be applied to the nonlinear battery model of Section II. For that, in Section III a transformation to a linear state estimation problem is proposed, where a prior knowledge of predominant nonlinearities in the model together with a coarse SOC estimate is utilized. Additionally, a mixed Kalman/H_∞ filtering method as well as a discrete-time SM- and an adaptive Luenberger (AL)-observer based approach is investigated for the model under consideration. Section IV presents the simulation framework as well as simulation results. The performance of the individual approaches is compared to state-of-the-art estimation schemes, like the reduced order SoC observer and the Coloumb counting method. Section V concludes the paper.

II. Battery model

In Fig. 1, the considered nonlinear battery model is presented. The battery runtime is modeled by the current-controlled current source I_b and the battery capacitance C_b. The controlled voltage source $V_\mathrm{OC}(SoC)$ describes the nonlinear relationship between the cell's SoC and its OCV. The normed voltage across the capacitance C_b corresponds to the

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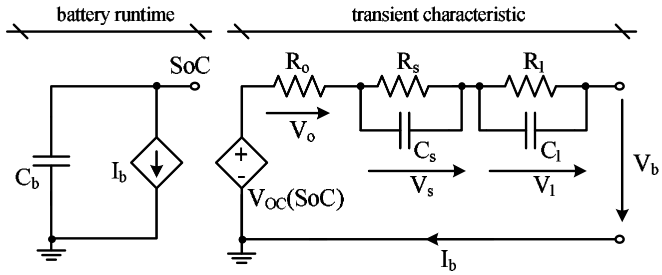

Fig. 1. Equivalent-circuit based cell model, adapted from [3, 11, 12].

SoC of the cell. R_s, C_s, R_l and C_l represent two parallel RC networks which are modeling both the short and the long time constant, which is present in a realistic battery step response [3]. V_s and V_l denote the corresponding voltages, R_o describes the ohmic part of the impedance, and V_b indicates the cell's terminal voltage. For simplicity, in this study we concentrate on the short time constant of the transient response and neglect the RC-network formed by R_l and C_l. Note that in general all model parameters are varying with the time, the SoC and the temperature.

III. State estimation

In this section, different estimation schemes will be applied to the battery model presented in Fig. 1, in order to estimate the SoC of a battery. In the following paragraphs, the application of Kalman, H_∞ and mixed Kalman/H_∞ filtering will be illustrated. Additionally, both a discrete-time SM observer and a discrete-time AL observer are shown to be applicable to the model under consideration. For completeness, in this section the concept of state-of-the-art estimation schemes like the reduced order SoC observer [13] or the CC method is furthermore briefly explained. These approaches will serve as a reference for the comparison presented in Section IV.

A. Coulomb counting method (CC-method)

By using the CC-method, the estimate of the SoC at time instant k, $\hat{SoC}_k$, is updated based on the relation

$$\hat{SoC}_k = \hat{SoC}_{k-1} + \frac{I_{b,k-1} \cdot \eta_{k-1} \cdot T_s}{C_{b,k-1}} \tag{1}$$

where the accuracy is mainly influenced by the estimate of the coulombic efficiency η_{k-1} during charge or discharge, as well as by the estimates for the cell's capacity $C_{b,k-1}$ and the initial SoC ($\hat{SoC}_0$). T_s indicates the sampling time. $\hat{SoC}_{k-1}$ defines the SoC at time instant $k-1$. For simplicity, in this work a perfect knowlegde of η is assumed, chosen as a constant scalar $\eta = 1$ for charge and discharge. Consequently, the charge or discharge current contributes to the charge or discharge process in a one-to-one relation. As the CC-method represents the most frequently used SoC estimation method in practice, the performance of this method serves as a reference in this work. Especially the influences of an erroneous knowledge of C_b or SoC_0, or an error-afflicted measurement of $I_{b,k-1}$ (e.g. due to an offset error or due to noise processes) are of particular interest in this work.

B. Estimation strategy

In order to counteract errors which are introduced by the use of the CC-method, in this paper the CC-method is combined with a battery model based approach. Based on the battery model presented in Fig. 1, we define a state vector $\mathbf{x}_k$, at time instant k, in the form of $\mathbf{x}_k = \begin{bmatrix} SoC_k & V_{s,k} \end{bmatrix}^T$. $V_{s,k}$ denotes the voltage across the RC-network formed by the elements R_s and C_s, at time instant k. By the application of the above mentioned state estimation schemes, an estimate of $\mathbf{x}_k$, denoted as $\hat{\mathbf{x}}_k = \begin{bmatrix} \hat{SoC}_k & \hat{V}_{s,k} \end{bmatrix}^T$, is determined. In this work, $\hat{\mathbf{x}}_k$ is used for the model-based correction of the CC-method. By deriving the discrete-time state space description of the battery model presented in Fig. 1, and by neglecting the influence of the long time constants of the battery transient, the time-varying state update equation results to

$$\mathbf{x}_k = \mathbf{F}_{k-1}\mathbf{x}_{k-1} + \mathbf{G}_{k-1}u_{k-1} + \mathbf{w}_{k-1} \tag{2}$$

with

$$\mathbf{F}_{k-1} = \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{-1}{C_{s,k}R_{s,k}}} \end{bmatrix}, \; \mathbf{G}_{k-1} = \begin{bmatrix} -\frac{T_s}{C_{b,k}} \\ 1 - e^{\frac{-1}{C_{s,k}R_{s,k}}} \end{bmatrix},$$

where $\mathbf{x}_{k-1}$ denotes the state vector at time instant $k-1$, and the system input is given by $u_{k-1} = I_{b,k-1}$. The noise samples in $\mathbf{w}_{k-1} \in \mathbb{R}^{2\times 1}$ correspond to a Gaussian noise process with a covariance matrix $\mathbf{Q}_{k-1} \in \mathbb{R}^{2\times 2}$ (process noise). The output equation of the derived state space description is given by

$$y_k = \tilde{\mathbf{H}}_k\mathbf{x}_k + \mathbf{L}_k u_k + V_{OC,k}(SoC_k) + v_k, \tag{3}$$

with $\tilde{\mathbf{H}}_k = \begin{bmatrix} 0, & -1 \end{bmatrix}$, $\mathbf{L}_k = [-R_{o,k}]$, furthermore $u_k = I_{b,k}$ and y_k denotes the battery voltage $V_{b,k}$ at time instant k. v_k represents a Gaussian measurement noise process, with a covariance matrix $\mathbf{R}_k \in \mathbb{R}^{1\times 1}$. Both the process and the measurement noise samples are assumed to be uncorrelated from one time step to the next. The term $V_{OC,k}(SoC_k)$ describes the cell's OCV at time instant k, for a given SoC_k [3, 11]. Usually, the OCV-SoC relation $V_{OC,k}(SoC_k)$ is assumed to be known in advance, based on the pre-characterization of a cell. In practical applications, a fixed OCV-SoC relationship is often assumed. This is justified by the fact that the OCV-SoC curves are almost independent of influence factors like the temperature or the aging. By splitting the OCV-SoC curve of the cell in both a linear and a nonlinear part, the term $V_{OC,k}(SoC_k)$ in (3) can be written as [13]

$$V_{OC,k} = b_1 \cdot SoC_k + b_0(SoC_k). \tag{4}$$

b_1 represents a constant parameter which describes the OCV's linear dependency on the SoC. $b_0(SoC_k)$ indicates the SoC dependent nonlinear residual term [13]. The reliability of the knowledge of $V_{OC,k}$ mainly depends on the accuracy with respect to the estimated SoC. By using (4), (3) can be rewritten as

$$y_k = \mathbf{H}_k \cdot \mathbf{x}_k + \mathbf{L}_k \cdot u_k + b_0(SoC_k) + v_k, \tag{5}$$

with $\mathbf{H}_k = \begin{bmatrix} b_1, & -1 \end{bmatrix}$. In practice, $R_{o,k}$, $R_{s,k}$, and $C_{s,k}$ are showing nonlinear dependencies on the SoC and the temperature. In this paper, we hand over the tracking of the battery model parameters to a simultaneous running parameter estimator. In this work, the model parameters are treated as both time-invariant and time-variant parameters, which are assumed to be known for further considerations. Exemplary concepts for the estimation of these time-varying parameters during runtime are treated in Section IV-A. Assuming a reliable a-priori knowledge of the model parameters, and by decoupling them from the state estimation process, the term $V_{OC,k}(SoC_k)$ represents the only nonlinear term in the considered state-space description which depends nonlinear on the SoC. Thus, in this work, the considered nonlinear estimation problem is transformed to a linear state estimation problem via the following proposed linearization method:

Due to an a-priori knowledge of the term $V_{OC,k}(SoC_k)$, also the parameters b_0 and b_1 are known in advance. In an embedded implementation, they can be stored in Look-up tables (LUTs) and intermediate values can be computed online by applying interpolation techniques [3]. In (4), $b_0(SoC_k)$ represents the nonlinear residual term of $V_{OC,k}$. By substracting an estimate of b_0 from the measured battery terminal voltage $V_{b,k}$, the linearized observation passed to the state estimation schemes reduces to

$$\tilde{V}_{b,k} = V_{b,k} - b_0(SoC_{CC,k}). \tag{6}$$

By the use of (6), the nonlinear equation (3) is transformed to a linear system output equation. To decouple the estimation of $b_0(SoC_{CC,k})$ from the state estimation, $b_0(SoC_{CC,k})$ is determined based on the SoC estimate $SoC_{CC,k}$. $SoC_{CC,k}$ denotes the SoC estimate obtained by the application of the CC-method. The linearized observation $\tilde{V}_{b,k}$ and the measured charging/discharging current $I_{b,k}$ serve as the inputs for state estimation. In the following subsections, the concepts of the applied battery state estimation schemes is presented in detail.

C. Discrete-time Kalman-filter

Based on the estimation strategy presented in section III-B, a discrete-time Kalman filter (KF) is applied. Thereby, the states of the linear dynamic system are estimated in a recursive fashion, based on noisy observations $V_{b,k}$. In general, the KF minimizes the expected value of the squared estimation error, given as

$$\hat{\mathbf{x}}_k = \underset{\hat{\mathbf{x}}_k \in \Re}{\arg\min}\, E\left[(\mathbf{x}_k - \hat{\mathbf{x}}_k)^T (\mathbf{x}_k - \hat{\mathbf{x}}_k) | \tilde{V}_{b,1}, \ldots, \tilde{V}_{b,k} \right], \tag{7}$$

whereby $\hat{\mathbf{x}}_k$ and $\mathbf{x}_k$ denote the true and the estimated state vector at time instant k, and $\tilde{V}_{b,1}, \ldots, \tilde{V}_{b,k}$ are the linarized system observations made up to time instant k. $E[\cdot]$ indicates the expectation operator. The equations of the KF algorithm are given by [14]

1) Initialization:

$$\hat{\mathbf{x}}_0^+ = E[\mathbf{x}_0] = E\left[[SoC_0, V_{s,0}]^T \right], \tag{8}$$
$$\mathbf{P}_0^+ = E\left[(\mathbf{x}_0 - \hat{\mathbf{x}}_0^+)(\mathbf{x}_0 - \hat{\mathbf{x}}_0^+)^T \right]. \tag{9}$$

2) A-priori estimation step:

$$\mathbf{P}_k^- = \mathbf{F}_{k-1} \mathbf{P}_{k-1}^+ \mathbf{F}_{k-1}^T + \mathbf{Q}_{k-1}, \tag{10}$$
$$\hat{\mathbf{x}}_k^- = \mathbf{F}_{k-1} \hat{\mathbf{x}}_{k-1}^+ + \mathbf{G}_{k-1} u_{k-1}, \tag{11}$$
$$\mathbf{K}_k = \mathbf{P}_k^- \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{R}_k)^{-1}. \tag{12}$$

3) A-posteriori estimation step:

$$\mathbf{P}_k^+ = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_k^-, \tag{13}$$
$$\hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_k^- + \mathbf{K}_k \left[\tilde{V}_{b,k} - (\mathbf{H}_k \hat{\mathbf{x}}_k^- + \mathbf{L}_k u_k) \right]. \tag{14}$$

$\hat{\mathbf{x}}_0^+$ denotes the exptected value of the initial state $\mathbf{x}_0$, $\mathbf{P}_0^+$ defines the uncertainity in $\hat{\mathbf{x}}_0^+$ and $\mathbf{I} \in \mathbb{R}^{2\times 2}$ denotes the identity matrix. If no knowledge of $\mathbf{x}_0$ is available, a reasonable choice for $\mathbf{P}_0^+$ is $\mathbf{P}_0^+ = \infty \mathbf{I}$. $\mathbf{P}_k^-$ and $\mathbf{P}_k^+$ denote the estimation error covariance matrices of the a-priori estimate $\hat{\mathbf{x}}_k^-$, or rather the a-posteriori estimate $\hat{\mathbf{x}}_k^+$. The Kalman gain $\mathbf{K}_k$ is adjusted based on the uncertainity in the a-priori state estimate. The a-posteriori estimate $\hat{\mathbf{x}}_k^+$ is updated based on $\mathbf{K}_k$, $\hat{\mathbf{x}}_k^-$ and the innovation term

$$i_k = \left[\tilde{V}_{b,k} - (\mathbf{H}_k \hat{\mathbf{x}}_k^- + \mathbf{L}_k u_k) \right]. \tag{15}$$

u_k is given by $u_k = I_{b,k}$. For the KF, $\mathbf{Q}_{k-1}$ and $\mathbf{R}_k$ denote noise covariance matrices which are ideally known. In practical applications, an exact knowledge of these covariance matrices is not necessarily given. Thus, in this work $\mathbf{Q}_{k-1}$ and $\mathbf{R}_k$ denote used-defined noise covariance matrices. The choice for $\mathbf{Q}_{k-1}$ and $\mathbf{R}_k$ influences the estimation performance.

D. Discrete-time H_∞-filter

Based on the estimation strategy presented in section III-B, the discrete-time H_∞-filter (minimax filter) is applied to the linear state estimation problem. It minimizes the effects of the worst possible disturbances (noises) and the worst setting of the initial state. Consequently, it minimizes the worst-case estimation error. Compared to the KF, no assumptions about the underlying noise processes are required. The cost function to be minimized, is given by [14]

$$J = \frac{\sum_{k=0}^{N-1} \|\mathbf{x}_k - \hat{\mathbf{x}}_k\|^2_{\mathbf{S_k}}}{\|\mathbf{x}_0 - \hat{\mathbf{x}}_0\|^2_{\mathbf{P}_0^{-1}} + \sum_{k=0}^{N-1} \left(\|\mathbf{w}_k\|^2_{\mathbf{Q}_k^{-1}} + \|\mathbf{v}_k\|^2_{\mathbf{R}_k^{-1}} \right)}. \tag{16}$$

$\mathbf{P}_0^{-1}$ and $\mathbf{S_k}$ are user-defined weighting matrices. By the choice of $\mathbf{S}_k$, $\mathbf{P}_0$, $\mathbf{Q}_k$ and $\mathbf{R}_k$, a-priori knowledge about the contribution of noise processes or uncertainties to the cost function may be included in the estimation process. $\hat{\mathbf{x}}_k$ is determined such that the cost function J is bounded below a user-defined bound θ, given as

$$J < \frac{1}{\theta}. \tag{17}$$

The applied H_∞-filter algorithm is given by [14, 15]

1) A-priori estimation step:

$$\hat{\mathbf{x}}_k^- = \mathbf{F}_{k-1} \hat{\mathbf{x}}_{k-1}^+ + \mathbf{G}_{k-1} \mathbf{u}_{k-1}, \tag{18}$$
$$\mathbf{A}_k = \left(\mathbf{I} - \theta \mathbf{S}_k \mathbf{P}_{k-1}^+ + \mathbf{H}_k^T \mathbf{R}_k^{-1} \mathbf{H}_k \mathbf{P}_{k-1}^+ \right)^{-1} \tag{19}$$
$$\mathbf{G}_k = \mathbf{F}_{k-1} \mathbf{P}_{k-1}^+ \mathbf{A}_k \mathbf{H}_k^T \mathbf{R}_k^{-1}. \tag{20}$$

2) A-posteriori estimation step:

$$\hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_k^- + \mathbf{G}_k \left[\tilde{V}_{b,k} - (\mathbf{H}_k \hat{\mathbf{x}}_k^- + \mathbf{L}_k u_k) \right], \tag{21}$$
$$\mathbf{P}_k^+ = \mathbf{F}_{k-1} \mathbf{P}_{k-1}^+ \mathbf{A}_k \mathbf{F}_{k-1}^T + \mathbf{Q}_{k-1}, \tag{22}$$

where $\mathbf{G}_k$ is called the H_∞ gain. θ has to be chosen such that the eigenvalues of the covariance matrix $\mathbf{P}_k^+$ have magnitudes less than one [14, 15].

E. Discrete-time mixed Kalman/H_∞ filtering

For the battery model under test, also the mixed Kalman/H_∞ filter estimation approach is applied. The mixed Kalman/H_∞ filtering approach provides an estimate which minimizes the Kalman cost function, among all estimators which are bound by the H_∞ filter performance bound. Thereby, the applied algorithm is written as [14]

$$\hat{\mathbf{x}}_k^+ = \hat{\mathbf{F}}_k \hat{\mathbf{x}}_k^- + \mathbf{G}_{k-1} u_k + \mathbf{M}_k \tilde{V}_{b,k}. \tag{23}$$

For stability, the matrices $\hat{\mathbf{F}}_k$ and $\mathbf{M}_k$ (gain) are chosen as

$$\hat{\mathbf{F}}_k = \mathbf{F}_{k-1} - \mathbf{M}_k \mathbf{H}_k \tag{24}$$

$$\mathbf{M}_k = \mathbf{P}_{a,k} \mathbf{V}_k^{-1}, \tag{25}$$

where $\mathbf{P}_{a,k}$ and $\mathbf{V}_k$ are defined by

$$\mathbf{P}_{a,k} = \mathbf{F}_{k-1}\mathbf{P}_{k-1}\mathbf{H}_k^T + \mathbf{F}_{k-1}\mathbf{P}_{k-1}(\frac{\mathbf{I}}{\theta^2} - \mathbf{P}_{k-1})^{-1}\mathbf{P}_k\mathbf{H}_k^T \tag{26}$$

$$\mathbf{V}_k = \mathbf{R}_k + \mathbf{H}_k\mathbf{P}_{k-1}\mathbf{H}_k^T + \mathbf{H}_k\mathbf{P}_{k-1}(\frac{\mathbf{I}}{\theta^2} - \mathbf{P}_{k-1})^{-1}\mathbf{P}_k\mathbf{H}_k^T. \tag{27}$$

The positive semidefinite matrix $\mathbf{P}_k$ is calculated in a recursive fashion, given as

$$\begin{aligned}\mathbf{P}_k = {} & \mathbf{F}_{k-1}\mathbf{P}_{k-1}\mathbf{F}_{k-1}^T + \mathbf{Q}_{k-1} - \mathbf{P}_{a,k}\mathbf{V}_k^{-1}\mathbf{P}_{a,k}^T \\ & + \mathbf{F}_{k-1}\mathbf{P}_{k-1}(\frac{\mathbf{I}}{\theta^2} - \mathbf{P}_{k-1})^{-1}\mathbf{P}_{k-1}\mathbf{F}_{k-1}^T.\end{aligned} \tag{28}$$

For the choice $\theta = 0$, the corresponding discrete-time algebraic Riccati equation (DARE) reduces to the DARE that is known from the discrete-time Kalman filter theory [14].

F. Reduced order SoC observer

Recently, in [13] a continuous-time reduced-order linear (ROL) SoC observer has been published, used as a reference in this work. There, both the CC-method as well as an OCV-based correction mechanism are combined to update the SoC. In this work, a discrete-time version of the ROL SoC observer is applied to the model under test, using the relation

$$\begin{aligned}S\hat{o}C_k = {} & S\hat{o}C_{k-1} - \frac{I_{b,k-1}T_s}{C_{b,k-1}} + \\ & L \cdot \left[V_{OC,k-1}(S\hat{o}C_{k-1}) - (V_{b,k-1} + V_{o,k-1} + V_{s,k-1})\right].\end{aligned} \tag{29}$$

L denotes a user-defined scalar observer gain. Note that even in the case of a perfect knowledge of $V_{s,k}$ and $R_{o,k}$, the ROL SoC observer is very sensitive to the choice of L.

G. Discrete-time adaptive Luenberger observer

By using the ROL SoC observer, a fixed user-defined observer gain L is used. Alternatively, in [8] an AL observer has been applied for state estimation of a linear battery model. Thereby, the observer gain $\mathbf{L}_k$ is adaptively adjusted based on Widrow's stochastic (approximative) gradient approach, using the instantaneous squared error between the estimated and the measured battery output voltage [8]. By using the stochastic gradient approach, the mean of $\mathbf{L}_k$ converges to the minimum mean squared error solution (optimal solution) of $\mathbf{L}_k$ [16]. In this paper, the AL observer is applied to the nonlinear battery model under test, given as

$$\mathbf{L}_k = \mathbf{L}_{k-1} + 2\eta i_k \mathbf{H}_{k-1}\mathbf{F}_{k-1}\frac{\delta\hat{\mathbf{x}}_{k-1}}{\delta\mathbf{L}}, \tag{30}$$

$$\hat{\mathbf{x}}_k = \mathbf{F}_{k-1}\hat{\mathbf{x}}_{k-1} + \mathbf{G}_{k-1}u_{k-1} + \mathbf{L}_k i_k, \tag{31}$$

$$\frac{\delta\hat{\mathbf{x}}_k}{\delta\mathbf{L}} = [\mathbf{F}_{k-1} - \mathbf{L}_k\mathbf{H}_{k-1}\mathbf{F}_{k-1}]\frac{\delta\hat{\mathbf{x}}_{k-1}}{\delta\mathbf{L}} + i_k\mathbf{N}. \tag{32}$$

η and $\frac{\delta\hat{\mathbf{x}}_k}{\delta\mathbf{L}}$ denote the learning rate and the partial derivative of $\hat{\mathbf{x}}_k$. In general, the used approximative gradient approach is expected to achieve a high accuracy for small values of η. If η is chosen too high, the stability of the AL observer cannot be guaranteed. $\mathbf{N} \in \mathbb{R}^{2\times 2}$ denotes an all-ones square matrix.

H. Discrete-time sliding mode (SM) observer

In [9, 10], continuous-time SM observers have been applied to the nonlinear model of a battery. There, the battery model has been assumed to be time-invariant. Based on [9, 10], we investigate a discrete-time SM observer for the time-variant battery model under test, given by

$$\begin{aligned}\hat{\mathbf{x}}_k = {} & \mathbf{F}_{k-1}\hat{\mathbf{x}}_{k-1} + \mathbf{G}_{k-1}u_{k-1} \\ & + \mathbf{h}_k i_k + \rho\Gamma\mathrm{sgn}(i_k).\end{aligned} \tag{33}$$

The switching gain ρ and the feedforward gain $\mathbf{h}_k$ are chosen such that the stability of the SM observer is guaranteed [9, 10, 17, 18]. $\mathrm{sgn}(i_k)$, the sign of the output reconstruction error, represents the discontinuous input for the feedback, chosen as $+1$ for $i_k > 0$, and -1 for $i_k < 0$. Thereby, $i_k = \mathbf{H}_k e_{x,k}$, where $e_{x,k}$ represents the state reconstruction error given by $e_{x,k} = \mathbf{x}_k - \hat{\mathbf{x}}_k$. $\mathbf{h}_k$ can be obtained by using the LQ method [9]. $\mathbf{h}_k$ is chosen as $\mathbf{h}_k^T = \mathbf{R}^{-1}\mathbf{H}_k\mathbf{O}_k$, to preserve stability. $\mathbf{R}$ and $\mathbf{O}_k$ denote positive definite matrices, where $\mathbf{O}_k$ is the solution of the corresponding discrete Riccati equation. $\mathbf{\Gamma}$ decomposes the process noise vector $\mathbf{w}_k$ as $\mathbf{w}_k = \mathbf{\Gamma}\xi$, where ξ denotes a bounded disturbance input [9].

I. Complexity

Computational effort and memory requirement are still primary criteria for embedded implementations. For Kalman/H_∞ filtering and the SM observer, time-invariant assumptions for the system and the noise covariances significantly ease the effort for an embedded implementation [14]. For the complexity of the other estimation schemes, we refer to [8, 13, 14].

IV. Simulation results

This section presents simulation results for the applied estimation schemes as well as the used simulation framework. Another subsection deals with model parameter estimation.

A. Parameter estimation

In practice, the model parameters R_o, R_s and C_s show dependencies on the SoC or the temperature. As they are used as input for the applied state estimation schemes, a reliable knowledge of these parameters is indispensable. For an estimation during runtime, it is referred to [19], where sequential least squares estimation schemes have been applied. Alternatively, the parameters may be determined based on pre-characterization of the battery under test. To estimate the ohmic part R_o of the impedance of the battery, instantaneous voltage/current changes may be utilized. For the estimation of C_b, two reliable SoC readings and the intermediate integrated charge may be used [20]. In this paper, the focus is on the analysis of the performance of the individual applied state estimation schemes. Thus, a perfect knowledge of the parameters R_o, R_s and C_s is assumed. In order to evaluate the performance of the applied state estimation methods for realistic use cases, both a time-invariant and a time-variant characteristic of the model parameters is considered. The uncertainty in C_b is treated below.

B. Simulation framework

To use the voltage $\tilde{V}_{b,k}$ and the current $I_{b,k}$ for state estimation, they are e.g. measured by an analog-to-digital converter (ADC). These are the only quantities which are directly measureable from the battery. $I_{b,k}$ serves as an input for the model-based state estimation, and $V_{b,k}$ represents the output of the considered battery model. In practice, uncertainties are existent on the input and the output of the model, as well as on the initial state estimate. E.g. for the KF the uncertainty in the initial estimate is modeled by $\mathbf{P}_0^+$. For simplicity, a perfect knowledge of the initial states has been assumed for all applied estimation schemes. For practical applications, we propose to use an OCV extrapolation method to cope with initial SoC uncertainities [3, 4]. The uncertainty in $\tilde{V}_{b,k}$ is modeled by the measurement noise v_k, which is assumed as a Gaussian noise process with the standard deviation $\sigma_{\tilde{V}_{b,k}} = 0.001$. For $I_{b,k}$, measurement errors and losses caused by the sampling process are modeled as a Gaussian noise process with a standard deviation of $\sigma_{I_{b,k}} = 0.0015$. This models worst-case errors around $\pm 3mV$ for $\tilde{V}_{b,k}$, and $\pm 6mA$ for $I_{b,k}$. Basically, uncertainties in a measurement are determined by the performance of the used measurement equipment. To emulate realistic conditions, $\sigma_{I_{b,k}}$ is chosen higher compared to $\sigma_{\tilde{V}_{b,k}}$, modeling tolerances of the employed sensing resistor. The uncertainties in the charge integration stage are modeled as

$$SoC_k = SoC_{k-1} + \frac{I_{b,k-1} \cdot T_s}{C_b} + w_{1,k-1}, \qquad (34)$$

where $w_{1,k-1}$ (first entry of $\mathbf{w}_k$) includes the uncertainty which is implicated by the not perfect knowledge of $C_{b,k}$. $w_{1,k-1}$ is assumed to be a Gaussian noise process, with a standard deviation of $\sigma_{w_{1,k-1}} = 0.0001$, simulating an accumulative worst-case SoC error of $\pm 0.03\%$ per iteration.

C. Simulation results

Subsequently, the simulation results of the applied state estimation schemes are presented. To evaluate the performance of the presented approaches, a periodic charge/discharge current profile with $t_{chg} = t_{dchg} = 5000s$ is used. A current rate of $1.1A$ is applied, followed by a period of rest of $1000s$, respectively, see Fig. 2 (a). The sampling time and the nominal capacity are chosen to $T_s = 1s$ and $C_b = 1.9Ah$. In literature, a common assumption is to use a time-invariant model for the battery. In this work, simulation results are presented with respect to both scenarios, a time-invariant (scenario 1) and a time-variant (scenario 2) system description. For scenario 2,

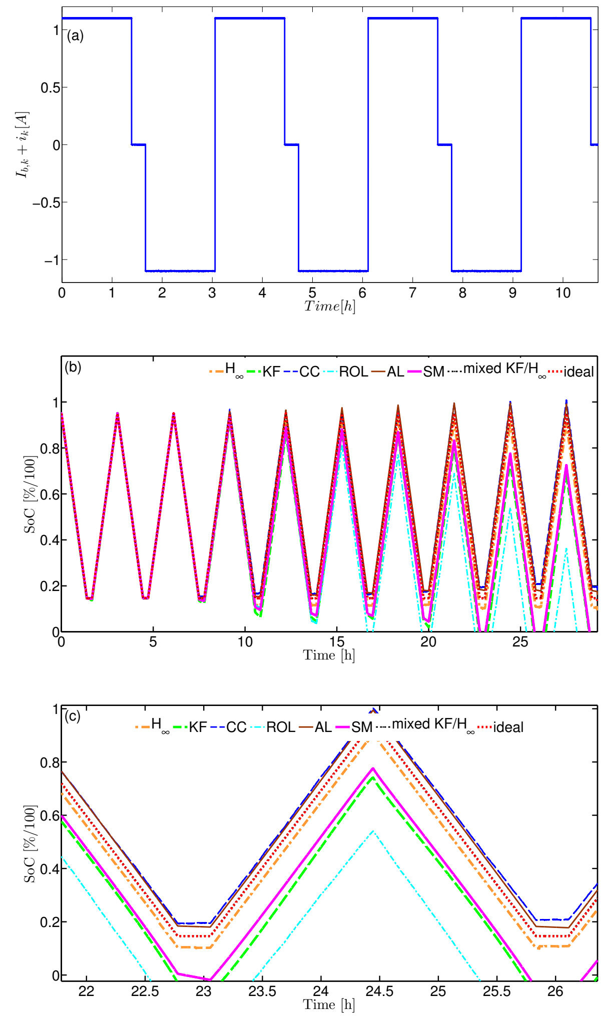

Fig. 2. Scenario 2: (a) Current pattern (b) SoC (10 cycles) (c) Zoomed SoC.

the parameters R_o, R_s and C_s are empirically chosen as

$$R_o = 0.1 + 0.28 \cdot \exp(-28.7 \cdot SoC), \qquad (35)$$

$$R_s = 0.08 + 0.13 \cdot \exp(-22.1 \cdot SoC), \qquad (36)$$

$$C_s = 685.3 - 402.9 \cdot \exp(-7.2 \cdot SoC). \qquad (37)$$

For scenario 1, (35), (36) and (37) reduce to their SoC independent terms. In practice, only a vague knowledge about the underlying noise processes is available. Thus, a pessimistic choice of the corresponding estimator noise variances is made. Thus, the covariance matrices for the Kalman/H_∞ filters are chosen as $\mathbf{Q}_{k-1} = \begin{bmatrix} 0.00012^2 & 0 \\ 0 & 0.0001^2 \end{bmatrix}$, $\mathbf{R}_k = \left[0.5^2\right]$, $\mathbf{S}_k = \mathbf{P}_0^+ = \mathbf{I}$. Moreover, an exact choice of the initial state variables is assumed. For the AL observer, a learning rate of $\eta = 1 \cdot 10^{-13}$ is chosen. The gain L of the ROL observer is chosen as $L = -1 \cdot 10^{-4}$. For the SM observer, the switching gain ρ and the vector $\mathbf{\Gamma}$ are chosen to $\rho = 10^{-4}$, $\mathbf{\Gamma} = \left[0.00012^2, 0.0001^2\right]^T$ and $\mathbf{R} = [1]$. The tuning parameter θ of the H_∞ and the mixed Kalman/H_∞ filter is adjusted to $\theta = 2 \cdot 10^3$. In Fig. 2 (b), the simulation results for the different applied estimation schemes are presented for 10 charge/discharge cycles. Thereby, the focus is on scenario 2,

TABLE I. COMPARISON: SOC ESTIMATION SCHEMES

Scenario 1	$\|e_{SoC,avg}\|[\%]$	$\|e_{SoC,wc}\|[\%]$
CC-method	1.1	2.6
KF	4.1	10
H_∞	0.2	2.1
Mixed KF/H_∞	0.01	0.03
ROL	3.7	11.4
AL	0.3	1.3
SM	2.7	6.9
Scenario 2	$\|e_{SoC,avg}\|[\%]$	$\|e_{SoC,wc}\|[\%]$
CC-method	1.2	2.3
KF	3.6	8.3
H_∞	0.2	1.85
Mixed KF/H_∞	0.01	0.03
ROL	3.5	9.8
AL	0.4	1.4
SM	2.4	5.8

for which a time-variant characteristic of the battery model parameters has been assumed. The red dashed line represents the ideal SoC curve, not affected by any measurement and/or modeling noise. In the beginning of the test, almost all lines are overlapping, yielding comparable performance. With ongoing time, especially the lines of the CC-method (blue dashed), the SM observer (magenta solid), the ROL observer (cyan dash-dotted) and the KF-method (green dashed) start to diverge from the ideal SoC. These methods are much more sensitive to the underlying noise processes, requiring a sophisticated fine-tuning adjustment. The ROL observer uses a fixed gain L, where a wrong adjustment results in a poor estimation performance. The lines for the H_∞ filter (orange dash-dotted), the mixed KF/H_∞ approach (black dotted) and the AL observer (brown solid) stay close to the ideal curve of the SoC, which indicates that these methods are able to deal with the introduced measurement and modeling noise. In Fig. 2 (c), a zoomed version of the plot in (b) is presented, focusing on the estimation error at the end of the applied test scenario. Thereby, the line for the mixed KF/H_∞ nearly overlaps with the curve of the ideal SoC. Consequently this approach yields the best performance for the presented test scenario. This is confirmed by the results presented in Table I. The table shows the magnitudes of the average and the worst-case SoC estimation errors, $|e_{SoC,avg}|$ and $|e_{SoC,wc}|$, for the first 3 charge/discharge cycles of the two applied scenarios (based on 20 independent simulation runs). Exhaustive simulations have shown the same performance trend for other applied load current profiles. In Table I, $|e_{SoC,avg}|$ is determined based on averaging over both the time and the number of total simulation runs. The value for $|e_{SoC,wc}|$ is identified by averaging the single-run worst-case estimation error over the total number of simulation runs.

V. CONCLUSION

In this work, the states of a nonlinear battery model have been estimated based on various applied estimation schemes. Based on the a-priori knowledge of pre-dominant nonlinearities, the problem has been transformed to a linear state estimation problem. The applied estimation schemes have been evaluated with respect to their ability to reduce the impact of error sources like measurement offsets or an inaccurate knowledge of the battery capacity. Already after some charge/discharge cycles, it turns out that the mixed Kalman/H_∞ estimation approach outperforms methods like Coulomb counting, Kalman- and H_∞- filtering, or approaches like the adaptive Luenberger observer, the sliding mode observer or the reduced-order SoC observer.

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